Taylor 展開覚書

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\begin{aligned}
f(x+h) &= f(x) + \int_0^h f'(x+t)\,dt \\
&= f(x) - \int_0^h (h-t)' f'(x+t)\,dt \\
&= f(x) - (h-t)f'(x+t)\Big|_0^h + \int_0^h (h-t) f''(x+t)\,dt \\
&= f(x) + hf'(x) - \int_0^h \left(\frac{(h-t)^2}{2!}\right)' f''(x+t)\,dt \\
&= f(x) + hf'(x) - \frac{(h-t)^2}{2!} f''(x+t)\Bigg|_0^h + \int_0^h \frac{(h-t)^2}{2!} f'''(x+t)\,dt \\
&= f(x) + hf'(x) + \frac{h^2}{2!} f''(x) - \int_0^h \left(\frac{(h-t)^3}{3!}\right)' f'''(x+t)\,dt \\
&= f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) - \int_0^h \left(\frac{(h-t)^4}{4!}\right)' f^{(4)}(x+t)\,dt \\
&= f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \cdots + \frac{h^n}{n!} f^{(n)}(x) + \int_0^h \frac{(h-t)^n}{n!} f^{(n+1)}(x+t)\,dt
\end{aligned}
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