演習問題 7

問題 1　次の方程式の解を求めよ。

(1)　$z^6 = 1$　$z^6 = \cos 2n\pi + i\sin 2n\pi$　すなわち、$z = \cos\dfrac{n\pi}{3} + i\sin\dfrac{n\pi}{3}$,　$n = 0,\ 1,\ 2,\ 3\ ,4\ ,5$

$\therefore z = 1,\ \dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i,\ \ -\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i,\ -1,\ -\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i,\ \dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i,$

(2)　$z^2 = 1 + \sqrt{3}\,i$　$z^2 = 2\left(\cos\left(\dfrac{\pi}{3} + 2n\pi\right) + i\sin\left(\dfrac{\pi}{3} + 2n\pi\right)\right)$　$n = 0,\ 1$　すなわち、

$z = \sqrt{2}\left(\cos\left(\dfrac{\pi}{6} + n\pi\right) + i\sin\left(\dfrac{\pi}{6} + n\pi\right)\right)$

$\therefore z = \dfrac{\sqrt{6}}{2} + \dfrac{\sqrt{2}}{2}i,\ -\dfrac{\sqrt{6}}{2} - \dfrac{\sqrt{2}}{2}i$

(3)　$z^3 = i$　$z^3 = \cos\left(\dfrac{\pi}{2} + 2n\pi\right) + i\sin\left(\dfrac{\pi}{2} + 2n\pi\right),\ n = 0,\ 1,\ 2$　すなわち、

$z = \cos\left(\dfrac{\pi}{6} + \dfrac{2n\pi}{3}\right) + i\sin\left(\dfrac{\pi}{6} + \dfrac{2n\pi}{3}\right)$

$\therefore z = \dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i,\ -\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i,\ -i$

(4)　$z^4 = -1$　$z^4 = \cos(2n+1)\pi + i\sin(2n+1)\pi,$　$n = 0,\ 1,\ 2,\ 3$ すなわち、

$z = \cos(\dfrac{2n+1)\pi}{4} + i\sin\dfrac{(2n+1)\pi}{4}$

$\therefore z = \dfrac{1}{\sqrt{2}} + \dfrac{i}{\sqrt{2}},\ \ -\dfrac{1}{\sqrt{2}} + \dfrac{i}{\sqrt{2}},\ \ -\dfrac{1}{\sqrt{2}} - \dfrac{i}{\sqrt{2}},\ \ \dfrac{1}{\sqrt{2}} - \dfrac{i}{\sqrt{2}}$

(5)　$z^2 = -1 + \sqrt{3}\,i$　$z^2 = 2\left(\cos\left(\dfrac{2\pi}{3} + 2n\pi\right) + i\sin\left(\dfrac{2\pi}{3} + 2n\pi\right)\right),$　$n = 0,\ 1$ すなわち、

$z = \sqrt{2}\left(\cos\left(\dfrac{\pi}{3} + n\pi\right) + i\sin\left(\dfrac{\pi}{3} + n\pi\right)\right),$

$\therefore z = \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{6}}{2}i,\ \ -\dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{6}}{2}i$