演習問題 6

問題 1　次の計算をせよ。

(1) $(\sqrt{3}+i)^{12} = 2^{12}\left(\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}i\right)^{12} = 2^{12}\left(\cos\dfrac{\pi}{6}+i\sin\dfrac{\pi}{6}\right)^{12} = 2^{12} = 4096$

(2) $\left(\sqrt{3}-i\right)^{-3} = 2^{-3}\left(\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}i\right)^{-3} = 2^{-3}\left(\cos\dfrac{\pi}{6}-i\sin\dfrac{\pi}{6}\right)^{-3} = 2^{-3}\left(e^{-\frac{\pi}{6}i}\right)^{-3} = 2^{-3}e^{\frac{\pi}{2}i} = \dfrac{i}{8}$

(3) $\left(1-\sqrt{3}i\right)^{6} = 2^{6}\left(\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i\right)^{6} = 2^{6}\left(e^{-\pi i/3}\right)^{6} = 2^{6}e^{-2\pi i} = 64$

(4) $(1+i)^{5} = 2^{5/2}\left(\cos\dfrac{\pi}{4}+i\sin\dfrac{\pi}{4}\right)^{5} = 2^{5/2}\left(\cos\dfrac{5\pi}{4}+i\sin\dfrac{5\pi}{4}\right) = -4(1+i)$

(5) $\left(\dfrac{\sqrt{3}}{2}+\dfrac{i}{2}\right)^{-9} = \left(\cos\dfrac{\pi}{6}+i\sin\dfrac{\pi}{6}\right)^{-9} = \left(e^{\pi i/6}\right)^{-9} = e^{-3\pi i/2} = i$