演習問題 5

問題 1　$\alpha = 1 + i,\ \beta = \dfrac{1 - \sqrt{3}\,i}{2}$ の時、次の問に答えよ。

(1)　$|\alpha|,\ |\beta|$ を求めよ。$|\alpha| = \sqrt{1^2 + 1^2} = \sqrt{2},\ |\beta| = \sqrt{\dfrac{1}{2^2} + \dfrac{3}{2^2}} = 1$

(2)　$\alpha,\ \beta$をそれぞれ極形式で表せ。

$$\alpha = \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right) = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$
$$\beta = \cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3} = \cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)$$

(3)　$\alpha\beta,\ \dfrac{\alpha}{\beta}$をそれぞれ極形式で表せ。

$$\alpha\beta = \sqrt{2}\left(\cos\frac{23\pi}{12} + i\sin\frac{23\pi}{12}\right) = \sqrt{2}\left\{\cos\left(-\frac{\pi}{12}\right) + i\sin\left(-\frac{\pi}{12}\right)\right\}$$
$$\frac{\alpha}{\beta} = \sqrt{2}\left\{\cos\left(-\frac{17\pi}{12}\right) + i\sin\left(-\frac{17\pi}{12}\right)\right\} = \sqrt{2}\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)$$

問題 2　$z = r(\cos\theta + i\sin\theta)$ のとき、次の計算を極形式で表せ。

(1)　$\bar{z} = r(\cos\theta - i\sin\theta) = r(\cos(-\theta) + i\sin(-\theta))$

(2)　$z + \bar{z} = r(\cos\theta + i\sin\theta) + r(\cos\theta - i\sin\theta) = 2r\cos\theta$

(3)　$z - \bar{z} = r(\cos\theta + i\sin\theta) - r(\cos\theta - i\sin\theta) = 2ir\sin\theta$

(4)　$z\bar{z} = r(\cos\theta + i\sin\theta)\cdot r\{\cos(-\theta) + i\sin(-\theta\}) = r^2$

(5)　$z^2 = r^2(\cos\theta + i\sin\theta)^2 = r^2(\cos 2\theta + i\sin 2\theta)$

(6)　$\bar{z}^2 = r^2(\cos\theta - i\sin\theta)^2 = r^2(\cos 2\theta - i\sin 2\theta) = r^2\{\cos(-2\theta) + i\sin(-2\theta)\}$

(7)　$\dfrac{1}{z} = \dfrac{1}{r}\dfrac{1}{(\cos\theta + i\sin\theta)} = \dfrac{1}{r}(\cos\theta - i\sin\theta) = \dfrac{1}{r}\{\cos(-\theta) + i\sin(-\theta)\}$

(8)　$\dfrac{1}{\bar{z}} = \dfrac{1}{r}\dfrac{1}{(\cos\theta - i\sin\theta)} = \dfrac{1}{r}(\cos\theta + i\sin\theta)$

(9)　$\dfrac{z}{\bar{z}} = \dfrac{(\cos\theta + i\sin\theta)}{(\cos\theta - i\sin\theta)} = (\cos\theta + i\sin\theta)^2 = \cos 2\theta + i\sin 2\theta$

(10)　$\dfrac{\bar{z}}{z} = \dfrac{(\cos\theta - i\sin\theta)}{(\cos\theta + i\sin\theta)} = (\cos\theta - i\sin\theta)^2 = \cos 2\theta - i\sin 2\theta = \cos(-2\theta) + i\sin(-2\theta)$