演習問題 4

問題 1 $\alpha = 2 + 2i,\ \beta = \sqrt{3} + i$ の時、次の問に答えよ。

(1) $|\alpha|,\ |\beta|$ を求めよ。 $|\alpha| = \sqrt{2^2 + 2^2} = 2\sqrt{2},\ |\beta| = \sqrt{3 + 1} = 2$

(2) $\alpha,\ \beta$をそれぞれ極形式で表せ。

$$\begin{aligned}
\alpha &= 2\sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right) = 2\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) \\
\beta &= 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)
\end{aligned}$$

(3) $\alpha\beta,\ \dfrac{\alpha}{\beta}$をそれぞれ極形式で表せ。

$$\begin{aligned}
\alpha\beta &= 4\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right) \\
\frac{\alpha}{\beta} &= \sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)
\end{aligned}$$

問題 2 $|\alpha| = 2,\ |\beta| = 3$ のとき、次の値を求めよ。

(1) $|\alpha\beta| = |\alpha||\beta| = 2 \times 3 = 6$ (2) $|\alpha^3| = |\alpha|^3 = 2^3 = 8$

(3) $\left|\dfrac{\alpha}{\beta}\right| = \dfrac{|\alpha|}{|\beta|} = \dfrac{2}{3}$ (4) $\left|\dfrac{\beta}{\alpha^2}\right| = \dfrac{|\beta|}{|\alpha|^2} = \dfrac{3}{4}$

問題 3 $z = 1 - i$ のとき、$\left|z - \dfrac{1}{z}\right|^2$ の値を求めよ。

$z - \dfrac{1}{z} = 1 - i - \dfrac{1+i}{(1-i)(1+i)} = 1 - i - \dfrac{1+i}{2} = \dfrac{1}{2} - \dfrac{3}{2}i$ だから、

$\left|z - \dfrac{1}{z}\right|^2 = \dfrac{1}{2^2} + \dfrac{3^2}{2^2} = \dfrac{10}{4} = \dfrac{5}{2}$

問題 4 次の複素数を極形式で表せ。

(1) $2i = 2\left(\cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}\right)$

(2) $-5 = 5\left(\cos\pi + i\sin\pi\right)$

(3) $-\sqrt{3} + i = 2\left(-\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i\right) = 2\left(\cos\dfrac{5\pi}{6} + i\sin\dfrac{5\pi}{6}\right)$

(4) $\dfrac{3\sqrt{3} + 3i}{2} = 3\left(\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i\right) = 3\left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)$