演習問題 14

問題 1 解の公式を用いて次の 2 次方程式を解け。

(1) $x^2+x+1=0$ $x=\dfrac{-1\pm\sqrt{1-4}}{2}=\dfrac{-1\pm\sqrt{3}\,i}{2}$

(2) $2x^2-5x+4=0$ $x=\dfrac{5\pm\sqrt{25-32}}{4}=\dfrac{5\pm\sqrt{7}\,i}{4}$

(3) $3x^2-7x+5=0$ $x=\dfrac{7\pm\sqrt{49-60}}{6}=\dfrac{7\pm\sqrt{11}\,i}{6}$

(4) $-2x^2+6x-7=0$ 両辺に -1 を掛けると、$2x^2-6x+7=0$ $\therefore x=\dfrac{3\pm\sqrt{9-14}}{2}=\dfrac{3\pm\sqrt{5}\,i}{2}$

(5) $x^2-10x+26=0$ $x=5\pm\sqrt{25-26}=5\pm i$

(6) $7x^2-6x+2=0$ $x=\dfrac{3\pm\sqrt{9-14}}{7}=\dfrac{3\pm\sqrt{5}\,i}{7}$

(7) $9x^2-6\sqrt{2}x+2=0$ $x=\dfrac{3\sqrt{2}\pm\sqrt{18-18}}{9}=\dfrac{\sqrt{2}}{3}$

問題 2 次の式を、複素数の範囲で因数分解せよ。

(1) x^2+4x-3 $=(x+2)^2-7=(x+2+\sqrt{7})(x+2-\sqrt{7})$

(2) $2x^2-2x+3$ $2x^2-2x+3=0$ を解くと、$x=\dfrac{1\pm\sqrt{5}\,i}{2}$

ゆえに、$2x^2-2x+3=2\left(x-\dfrac{1+\sqrt{5}\,i}{2}\right)\left(x-\dfrac{1-\sqrt{5}\,i}{2}\right)$

(3) x^2+8x+5 $=(x+4)^2-11=\left(x+4+\sqrt{11}\right)\left(x+4-\sqrt{11}\right)$

(4) $2x^2+3x+2$ $2x^2+3x+2=0$ を解くと、$x=\dfrac{-3\pm\sqrt{9-16}}{4}=\dfrac{-3\pm\sqrt{7}\,i}{4}$

$$\begin{aligned}\text{ゆえに、}2x^2+3x+2 &= 2\left(x-\frac{-3+\sqrt{7}\,i}{4}\right)\left(x-\frac{-3-\sqrt{7}\,i}{4}\right)\\ &= 2\left(x+\frac{3-\sqrt{7}\,i}{4}\right)\left(x+\frac{3+\sqrt{7}\,i}{4}\right)\end{aligned}$$